# An Application of Fractal Geometry to Porosity Modeling

by

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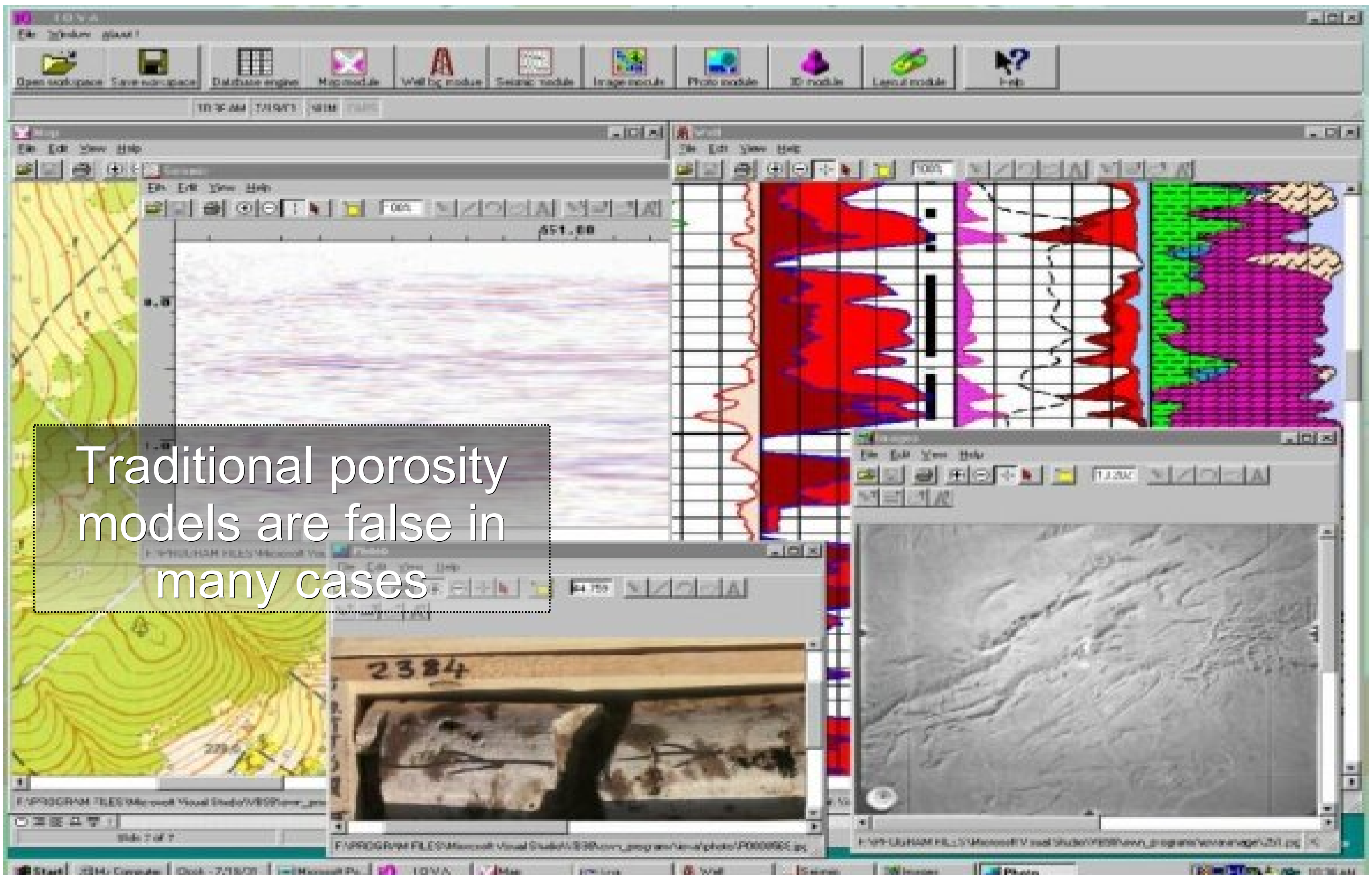
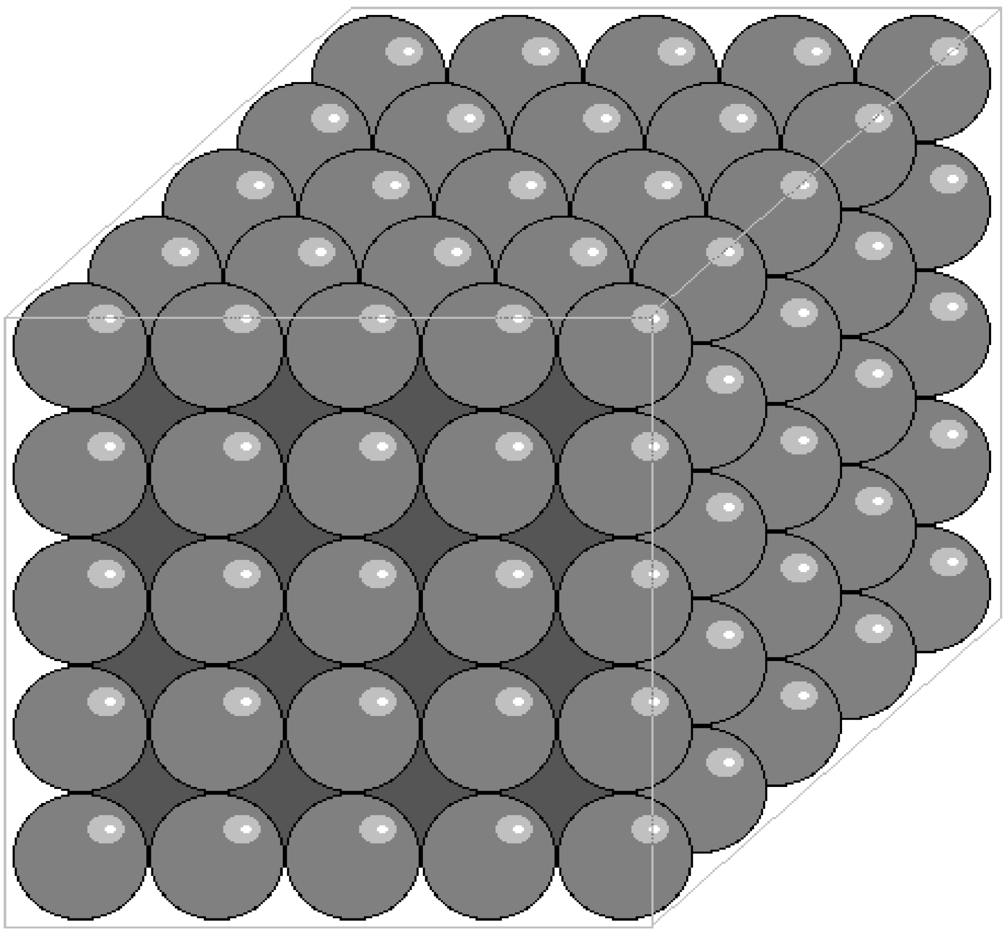
# Traditional models

Well log analysts – Reservoir geologists – Production experience




Traditional porosity models are false in many cases

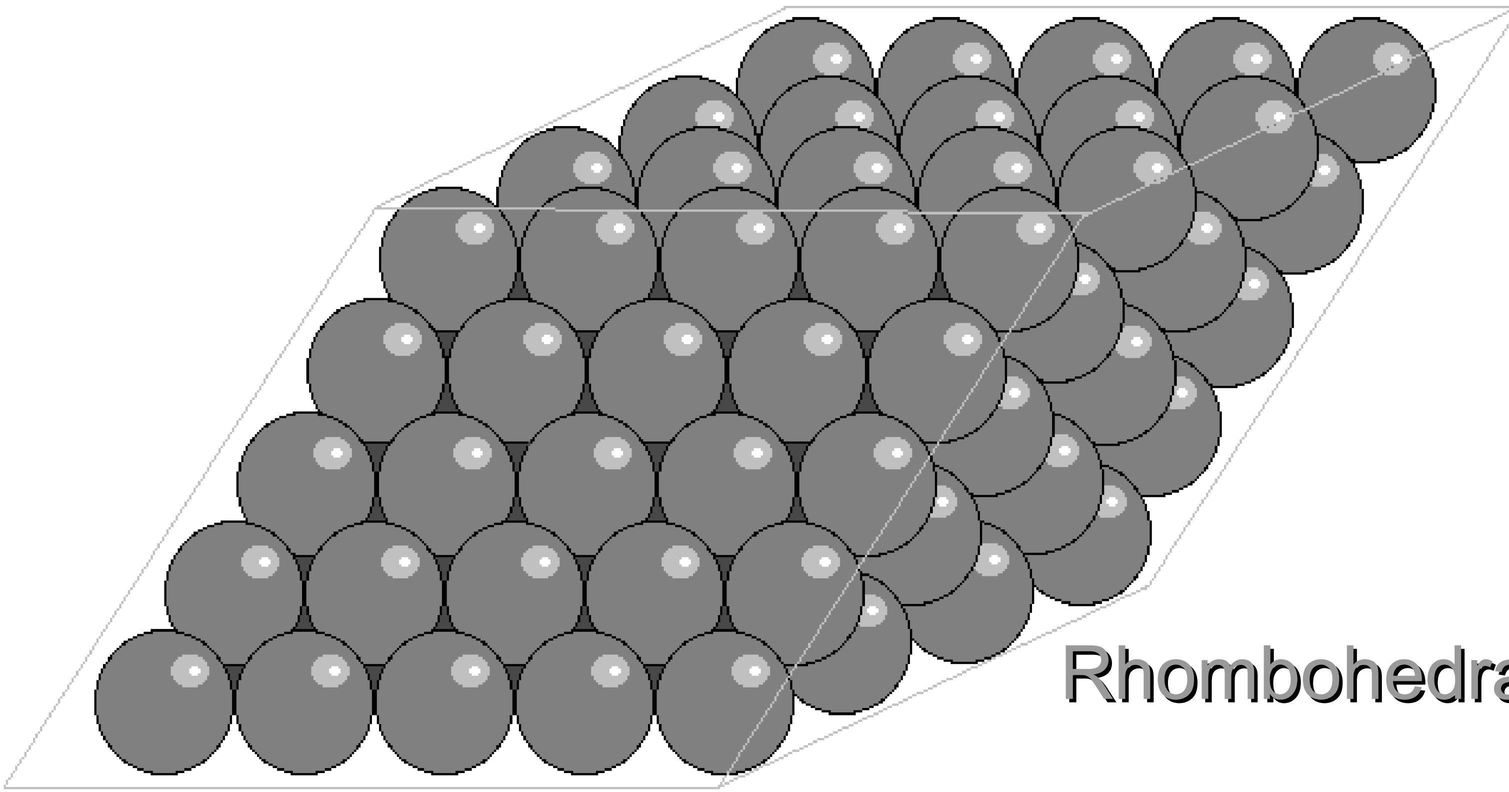
# Traditional models



## Ideal arrangement:

- Small and uniform spheres
- Mechanically unstable
- Porosity is 47.6%, unrealistic

# Traditional models

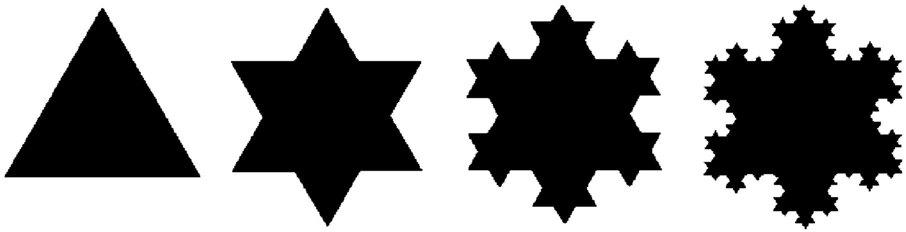


Rhombohedral arrangement:

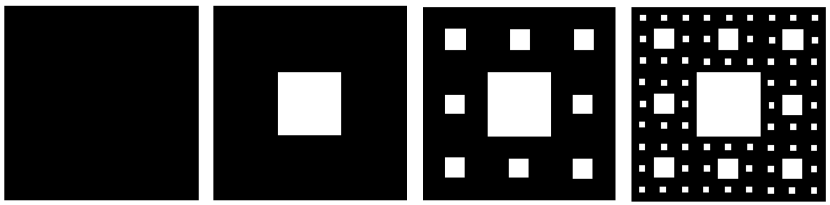
- Small and uniform spheres
- Mechanically stable configuration
- Porosity is 26%, realistic

# Strange fractals

Koch curve



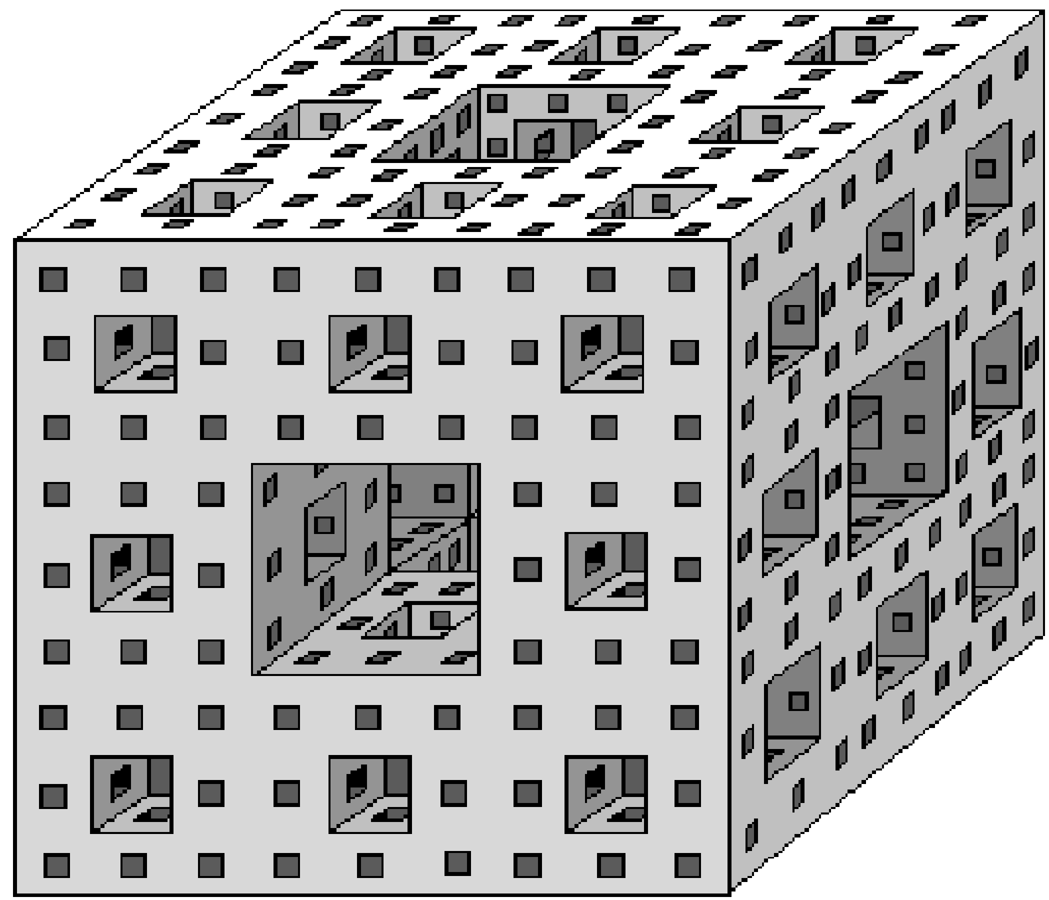
Sierpinsky carpet



etc ...

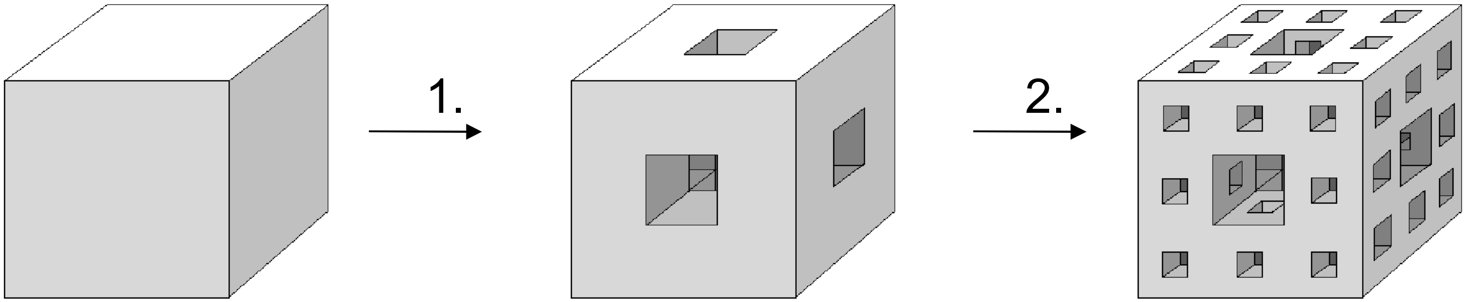
# Fractals and petrophysics

Menger sponge:

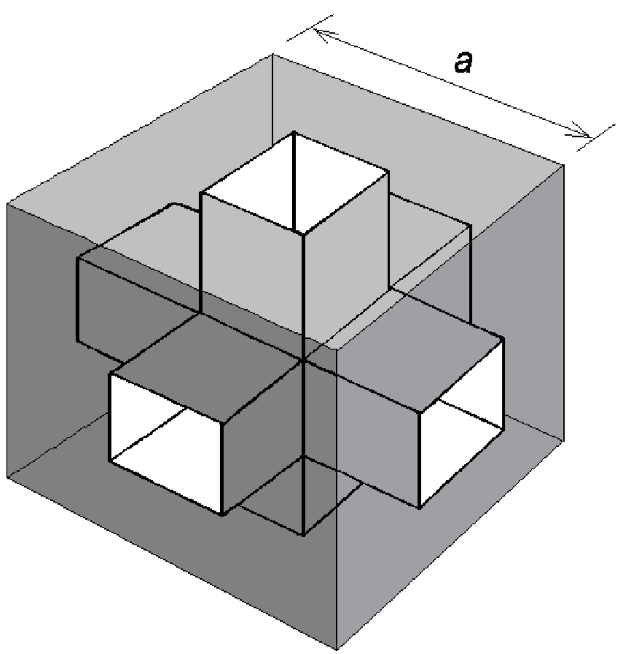


- strong structure
- high porosity values

# Fractals and petrophysics




1.

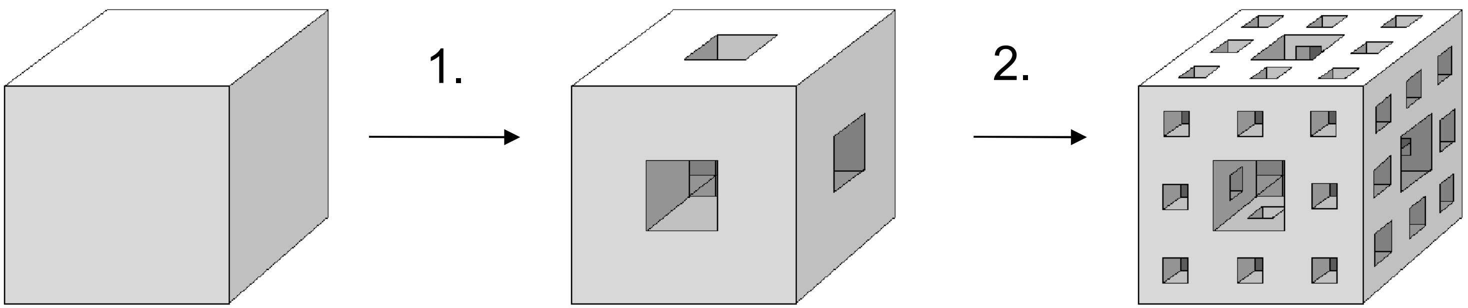



$V \quad = a^3$ , pore volume $V_p = 7(a/3)^3$

$V_{bulk} = a^3 - 7(a/3)^3 = 20(a/3)^3$

$\phi_1 \quad = V_p / V = 7(a/3)^3 / 27(a/3)^3 = 0.26$

# Fractals and petrophysics




2. $\phi_2 = V_p/V = 7/27 + 20 * (7/27)/27 = 0.45$

.

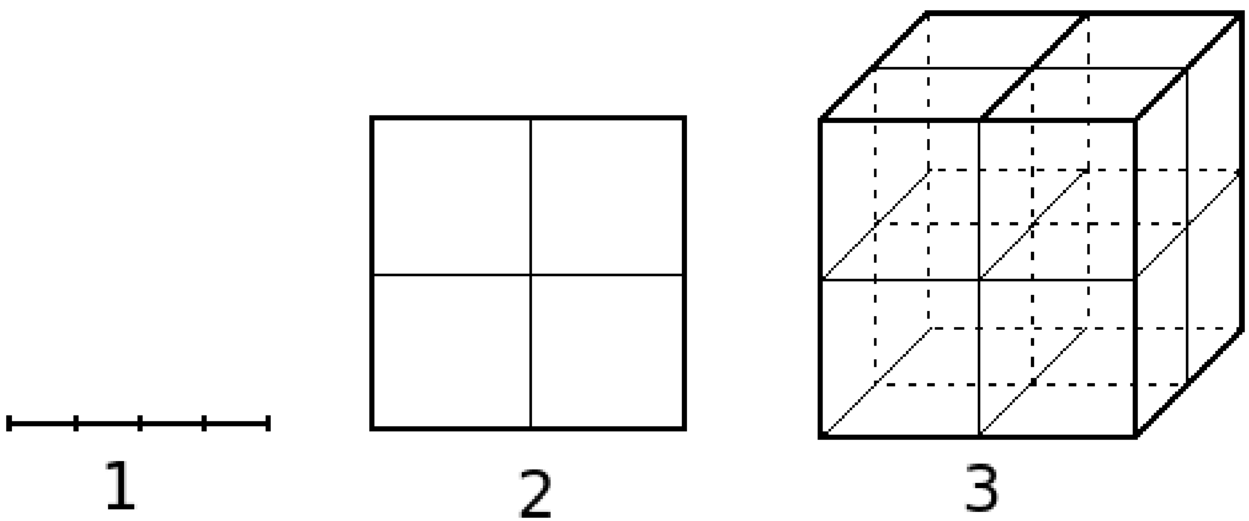
. $$V_p = \sum_{i=1}^{n} V_i ,$$

.

n $\phi_n = V_p/V = \frac{\sum_{i=1}^{n} V_i}{V}$ while $n \to \infty$, then $\phi \to 1$.

# Fractals and petrophysics

## Fractal dimension




Let us divide objects into ‚N' congruent parts

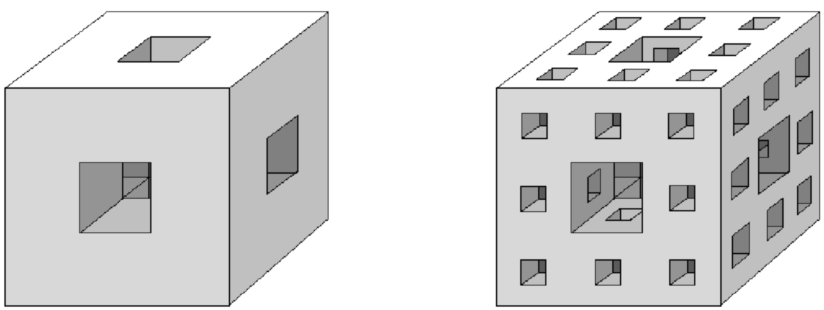
‚r ' is the similarity rate

$$D = \frac{log N}{log(1/r)}$$

1. Line: N=4, r=1/4, D = log 4 / log 4 =1
2. Square: N=4, r=1/2, D = log 4 / log 2 = 2
3. Cube: N=8, r=1/2, D = log 8 / log 2 =3

# Fractals and petrophysics

## Fractal dimension for Menger sponge



Let us divide objects into ‚N' congruent parts

‚r ' is the similarity rate

$$D = \frac{log N}{log(1/r)}$$

N = 20, r = 1/3, D = log 20 / log 3 = 2.7268

# Fractals and petrophysics

## Fractal dimension for Koch curve

Let us divide objects into ‚N' congruent parts

‚r ' is the similarity rate

1. N = 4,   r = 1/3,   D =  log 4 / log 3   = 1.261
2. N= 16,   r = 1/9,   D =  log 16 / log 9 = 1.261

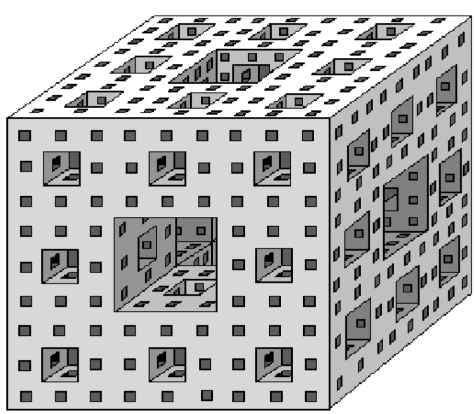
$$D = \frac{log N}{log(1/r)}$$

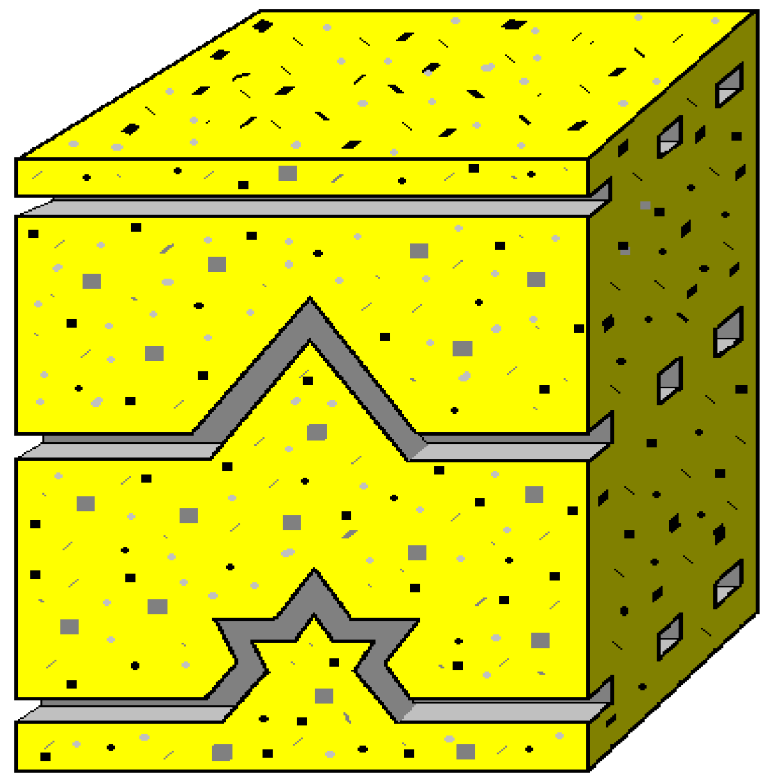
# Fractals and petrophysics

## Let us create fractal based synthetic rock models

for example:

Realize, that any kind of synthetic rock can be constructed based on fractals, such as Koch curve, Menger sponge or another ones.

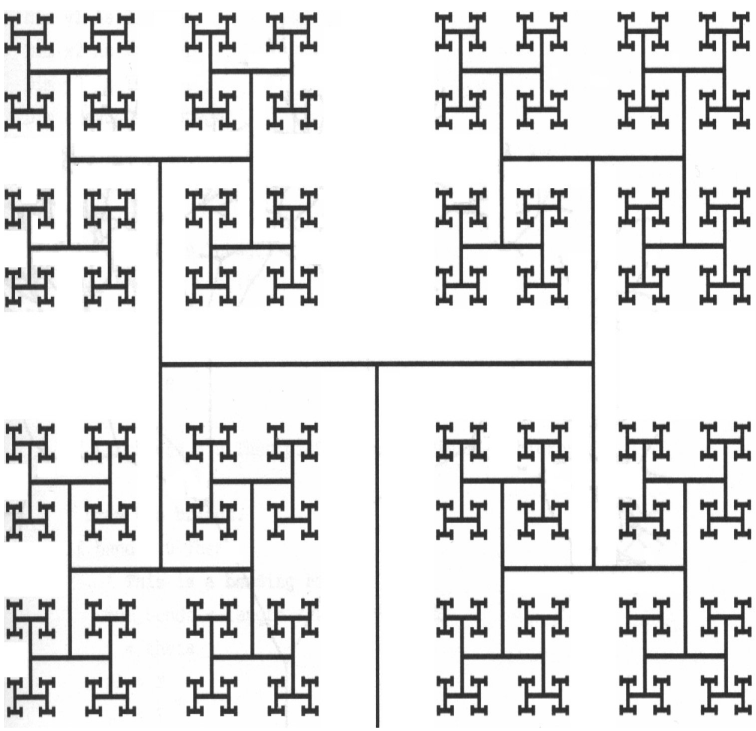


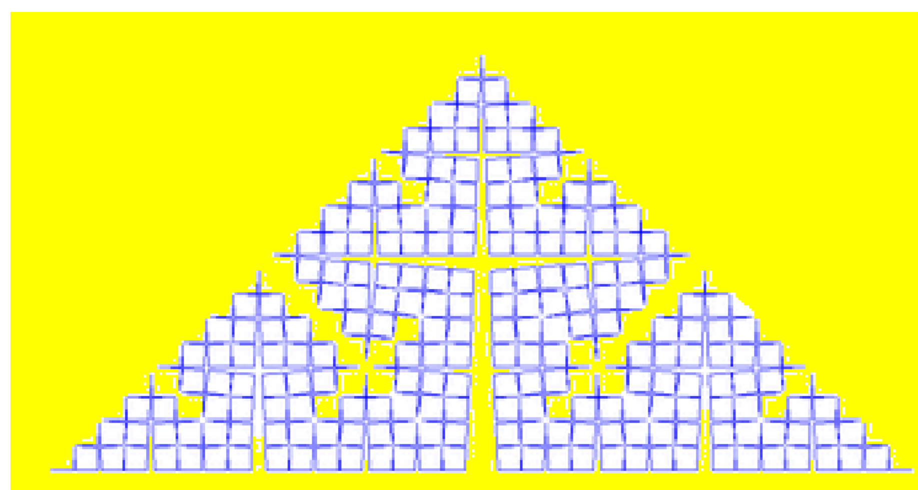


# Fractals and petrophysics

Different rock types can be modelled with different fractals,

some possible fractured rocks

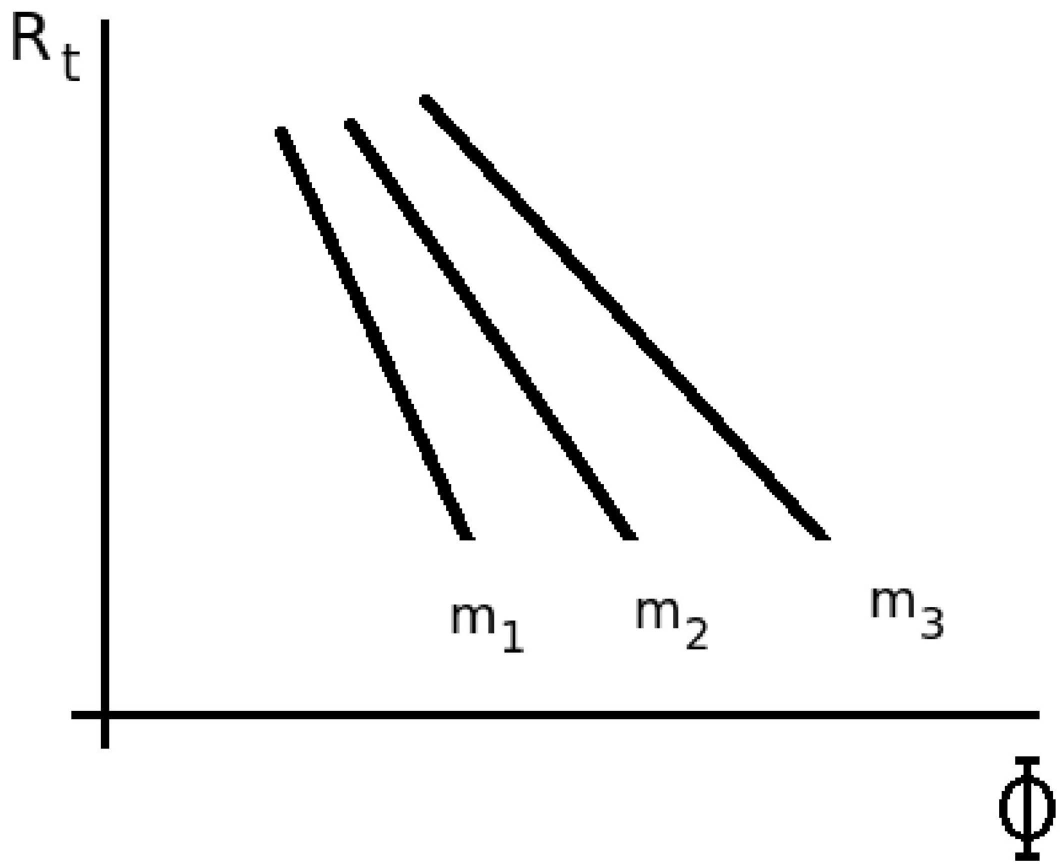




# Next steps

- Comparison with core lab measurements
- Define fractals which describe many rock types with various pore space
- Fractured rocks and carbonates are in the focus

# Fractals and petrophysics




$$R_t \sim \frac{1}{\phi^m}$$

Archie formula

$$R_t = \frac{R_w}{\phi^m S_w^n}$$

where

- $R_t$ is the true resistivity
- $R_w$ is the brine resisitivity
- $S_w$ is the water saturation
- $\phi$ is the porosity
- m is the cementation exponent
- n is the saturation exponent

# Fractals and petrophysics

## Another formula for fractal dimension

$$N(r) \sim \frac{1}{r^{-D_f}}$$

- where ‚r' is the radius (or characteristic length) of a unit chosen to fill the fractal object,
- ‚N(r)' is the number of the units within a radius of ‚r', required to fill the entire fractal object,
- and ‚$D_f$' is the fractal dimension