Triangulation With Smoothing

Triangulation is a process of grid generation that is most commonly applied to data that requires no regional averaging, such as elevation readings. The surface created by triangulation passes through (honours) all of the original data points while generating some degree of "overshoot" above local high values and "undershoot" below local low values. Elevation is an example of point values that are best "surfaced" with a technique that predicts some degree of over- and underestimation. In trying to model a topographic surface from scattered elevation readings, it is not reasonable to assume that data points were collected at the absolute top or bottom of each local rise or depression in the land surface. This is especially true in modeling subsurface layers using elevation readings from bore hole data.
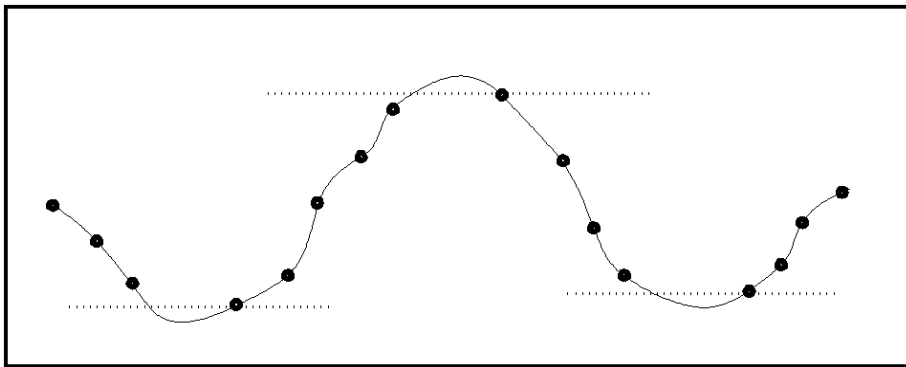

Figure 3.1. Using triangulation, the interpolated surface passes through the original data points, however, peaks and valleys will extend beyond the local maximum and minimum values.

Triangulation involves a process whereby all original data points are connected in space by a network of triangular faces, drawn as equilateral as possible, forming what is referred to as a Triangular Irregular Network (TIN) shown in Figure 3.2. Points are connected based on a nearest neighbour relationship (the Delaunay criterion) which states that a circumcircle drawn around any triangle will not enclose the vertices of any other triangle.

Figure 3.2. An example of the triangular irregular network (TIN)

A smooth grid surface is then fitted to the TIN using a bivariate fifth-order polynomial expression in the X and Y direction for each triangle face. This method guarantees continuity and smoothness of the surface along the sides of each triangle and smoothness of the surface within each triangle. The slope blending algorithm is designed to calculate new slope values for each of the triangle vertices (i.e., each point of the data set) where the influence of adjacent slopes in the blending calculation are weighted according to specified triangle properties.

Five properties of data point geometry and value greatly influence the ability of the slope blending algorithm to control smoothing of the TIN surface. These include: a) triangle centroid location, b) triangle aspect ratio, c) triangle area, d) angle versus slope of the triangle, and e) statistically-derived slope of a triangle vertex. For example, triangles with centroids farther from the vertex being solved have less influence on the slope calculation than triangles whose centroids are closer, or triangles with greater areas have greater influence in the slope calculation than triangles with a smaller area. The end result is a smoothing process that significantly reduces the frequency of angular artifacts, representing remnants of the original TIN facets, in the final gridded surface. For an instructional lesson on TIN interpolation refer to Lesson 3 of the *Installation and Tutorials* manual.

The Select Table and Column Dialogue

As discussed in the previous section, the *Select Table and Column* dialogue box is common to all interpolation methods initiated from the *Select Interpolation Method* screen of the Interpolation Wizard. The only difference being that, in accessing this dialogue using the Triangulation with Smoothing method, the user is presented with one additional option that is normally greyed.
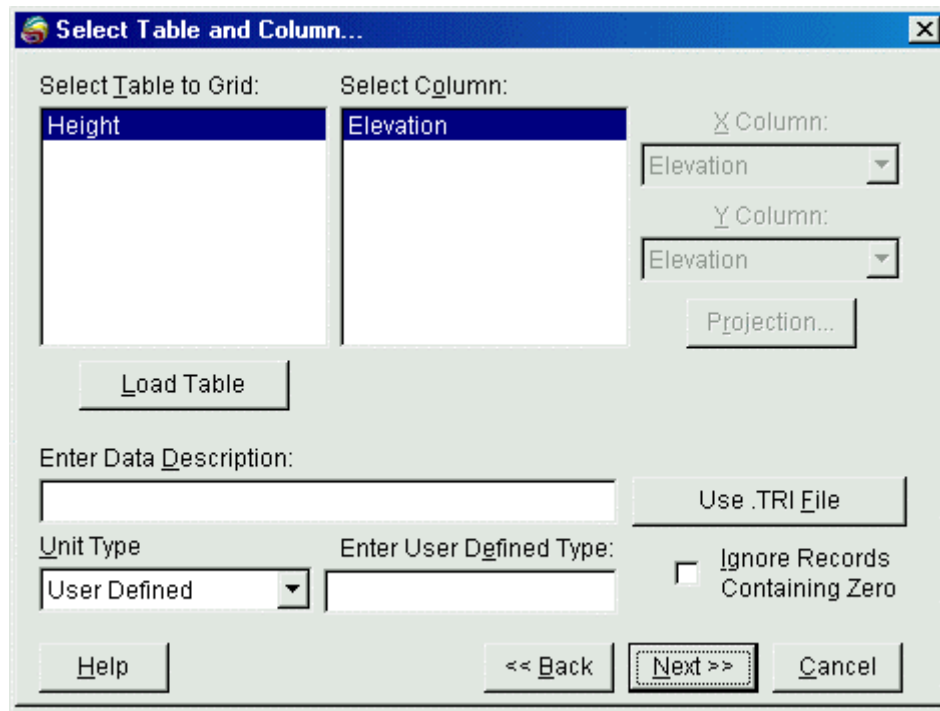

1   The *Use .TRI File* button located in the bottom right corner of the dialogue allows users to build grid files directly from an existing TIN file(.TRI). Selecting this button prompts the user to select a .TRI file and then moves directly to the *TIN Interpolation* dialogue, skipping the triangulation step.

The TIN Interpolation: Create TRI Dialogue

Once the data points have been analyzed by *Vertical Mapper* and exported to a temporary file, the *TIN Interpolation: Create TRI* dialogue box appears. Two features of the triangulation algorithm, designed to build the Triangular Irregular Network (TIN) surface, are particularly critical and should be carefully considered.

1.   Maximum Triangle Side Length

2.   Coincident Point Distance
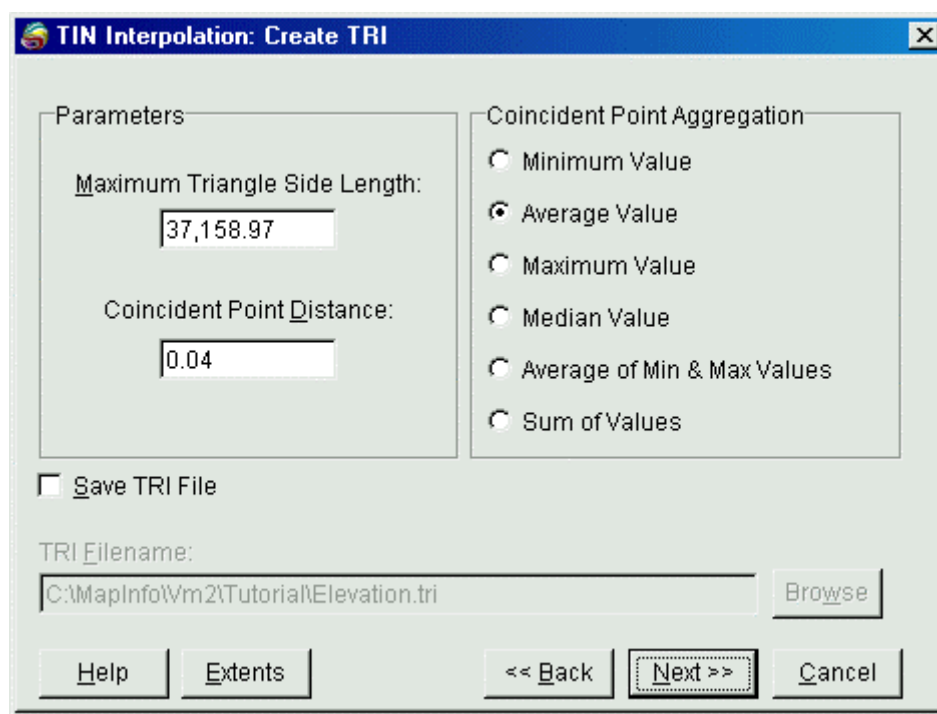
1   ***Maximum Triangle Side Length*** setting is used to limit the creation of triangles between points that the user considers too far apart. Depending upon the geographic distribution of points in a file, this distance setting will restrict the generation of unnecessarily large and/or sliver triangles and will limit interpolation between points across distances that may be inappropriate. For example, a set of field observations may contain large gaps in the coverage due to sampling problems. If the gaps are quite large, it would be inappropriate in most cases to interpret new values lying within these areas. Generally the user is expected to have an understanding of the distribution characteristics of their own data in order to enter an appropriate value.

2   ***Coincident Point Distance*** setting is a method of grouping or aggregating data points into a single new point with a recalculated value. The setting can be used in one of two ways. To prevent the creation of very small triangles, choose a small or zero distance to treat points in the data set which are very closely spaced or which overlie each other as a single point. As the distance setting becomes greater, the number of points found within each circular area will correspondingly increase. This may be appropriate in dealing with a highly variable and irregularly distributed data set. The ***Coincident Point Aggregation*** setting allows the user to define the mathematical expression for handling aggregated data. For example, choosing a large coincident point distance and selecting *Use Average Value* will result in the creation of a new set of data points for triangulation, spaced approximately according to the distance setting and with recalculated values based on the average of all points in each coincident point area. New points are placed at the geometric centre of the original group.

> ***Tip:*** Use the Ruler tool in MapInfo to examine your point data file and measure gaps in the distribution of points over the map area.   Use this information as a guideline in entering an appropriate value for the *Maximum Triangle Side Length*.

- In this dialogue, the user may check the ***Save TRI File*** box to save the TIN surface for future gridding. A file name for the .TRI file can be entered in this dialogue box.

- The user may either select the *Next >>* button to proceed or, if modifications to the previous dialogue are required, select the *<< Back* button to return to the *Select Table and Column* dialogue.

The TIN Interpolation Dialogue

The grid creation process consists of overlying a regularly-spaced network of equal-sized cells on top of the triangulated (TIN) surface and then estimating a new value for each cell based on a combination of variables. The variables include original point values, slope of individual and adjacent TIN faces, and distance to neighbouring triangles. The end result is a smoothed surface that passes through all of the original data points.

- The *TIN Interpolation* dialogue allows a number of user-defined parameters to be set in the algorithm that controls grid generation from the TIN surface.
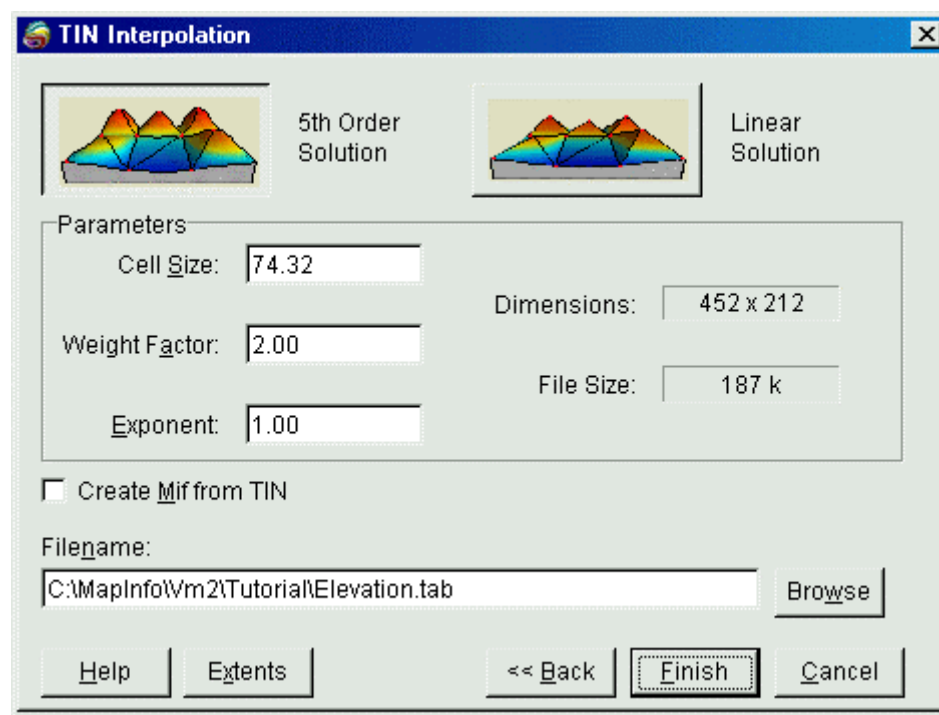

1   *Cell Size* is set in real units. The value chosen should be a compromise between the degree of resolution required for analysis and visualization purposes and the processing time and file size. The default value is calculated by dividing the diagonal extent of the point file by 500 which is considered an optimum number based on the average computing power available.

2   *Weighting Factor* defines the degree of influence imposed by neighbouring triangles in calculating a slope value for each triangulation point. Increasing the weighting factor will increase the relative influence of more distant triangle faces and will result in a greater degree of smoothing of the derived surface. Values range between one and one hundred.

3   *Exponent* is the variable that defines the exponential decay of influence of neighbouring triangle faces the farther they lie from each point. Increasing the exponent will decrease the relative influence of more distant triangles. Values range between one and twenty.

1. The mathematical expression used to estimate or interpolate Z- values from the triangle slope data at individual grid nodes is controlled by one of two *Solution Methods*. The graphical icon

associated with each method is designed to illustrate the manner in which each solution calculates new grid values from the TIN surface.
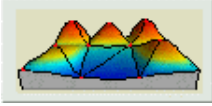
 A **Linear Solution** calculates the grid values **directly** from the TIN surface, therefore no derivative slope solution is applied. The result is a grid surface that exactly duplicates the angular appearance of the TIN.
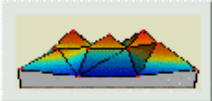
 A **Fifth Order Solution** applies a complex polynomial expression to the calculation of each grid node value. The calculation is based on solving for a number of slope derivatives. The result is a more highly smoothed surface that displays minor angular artifacts from the original TIN.

4   Select **Create MIF From TIN** check box to save the .TRI file as a MapInfo format region file (.MIF). The triangulated surface may then be viewed in a map window as a wireframe representation using the *Table > Import* command in MapInfo.

Tip: The default values for the weighting factor and the exponent are appropriate for a majority of point data sets. However, it is suggested that the user experiment with various settings in order to evaluate which variable generates the most appropriate representation of their own data.

- The user is prompted to enter a file name for the new grid in the **Filename** area.

1   The **Extents** button opens an information box that summarizes the geographic size and the Z-value range of the original point data base, the density of the points, as well as the data value units.
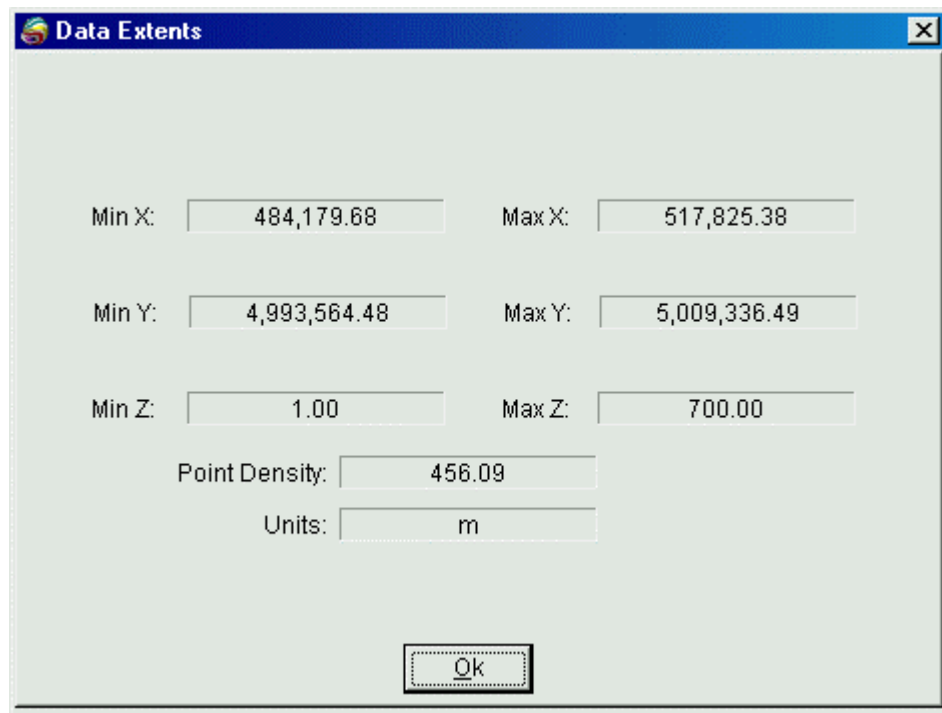
- The user may either select the **Finish** button to complete the gridding process or, if modifications to the previous dialogue are required, select the **<< Back** button to return to one or more dialogues back. Once the grid is created, it appears in a MapInfo Map window with a default colour palette applied. The user may change the colour range assigned to the grid file using commands discussed in Chapter 7.