MANIPULATIVE MAP PROJECTIONS: LINKING PROJECTION CONCEPTS TO THE BARBARA PETCHENIK COMPETITION

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"Creativity, as has been said, consists largely of rearranging what we know in order to find out what we do not know. ...Hence to think creatively we must be able to look afresh at what we normally take for granted."

-- George Kneller from ***The Art and Science of Creativity*** *(1965).*

Introduction

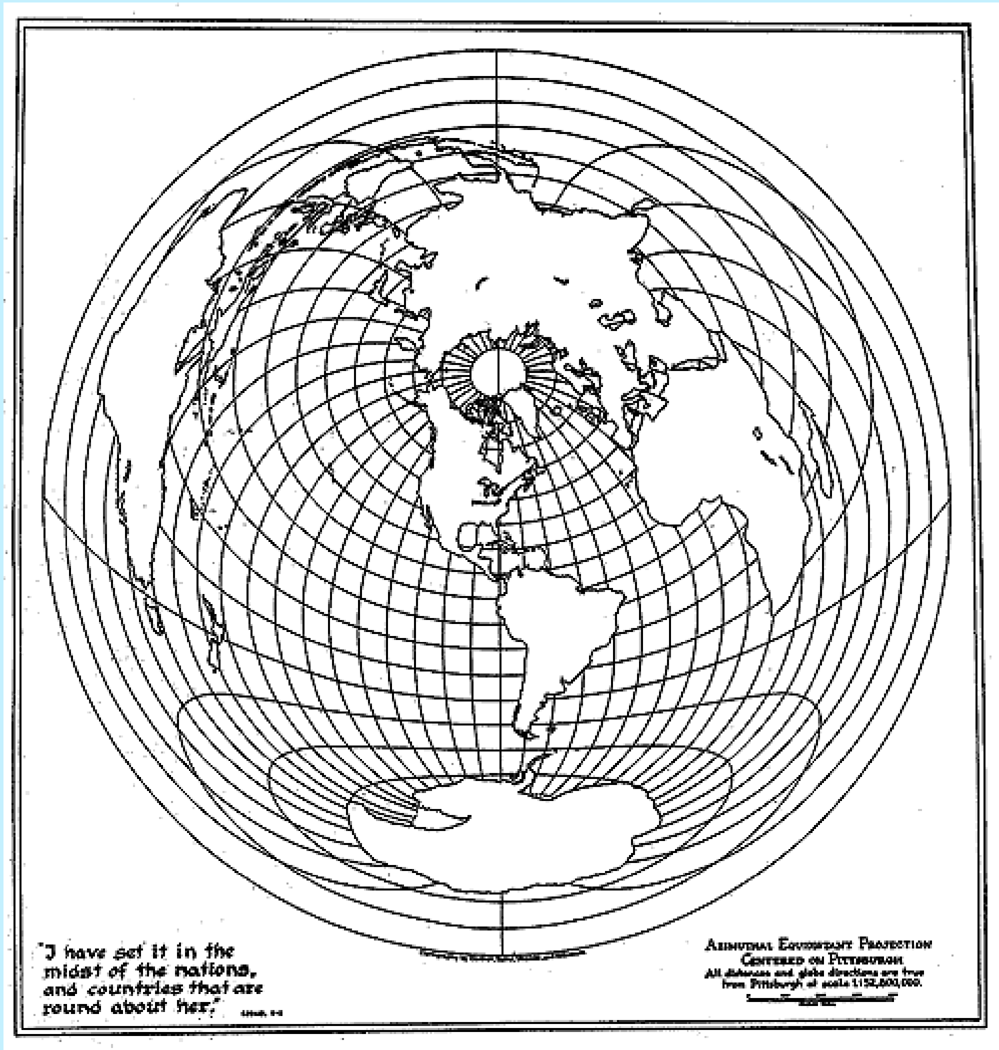

Figure 1: An Azimuthal Equi-distant projection centered on Pittsburgh, PA, USA.

I do not know when I first became interested in map projections but it may have been in graduate school when I first heard the phrase "**rectifying the globe**" -- positioning the globe in such a way that it replicates one's own position on and view of the earth. This seemed a more rational way of contemplating the earth than from a place out on the ecliptic, a place where few of us have ever been, where the earth sits rigidly at a 23 1/2 degree tilt. About the same time I discovered Richard Edes Harrison's 5 foot diameter nomograph which allowed me and three other graduate students to make an azimuthal equidistant projection of the world centered on Pittsburgh, PA (Figure 1). With that I came to understand how I could "rectify" the world on a map.

Figure 2: An Oblique Mollweide projection with equal area spherical triangles superimposed. From Fisher and Miller (1944, 75).

Since then I have been intrigued by oblique projections in various orientations (Figure 2) and appalled by how monotonously we present the earth, so often in its equatorial aspect.(1) We certainly don't seem to have much fun exploring the infinite number of ways that we can present the earth. Even with all the new computer programs now available, I am incredulous at how many seem limited to equatorial aspects of dozens of very similar projections and often neglect their oblique aspects or the perspective projection which offers the greatest improvisational opportunities.

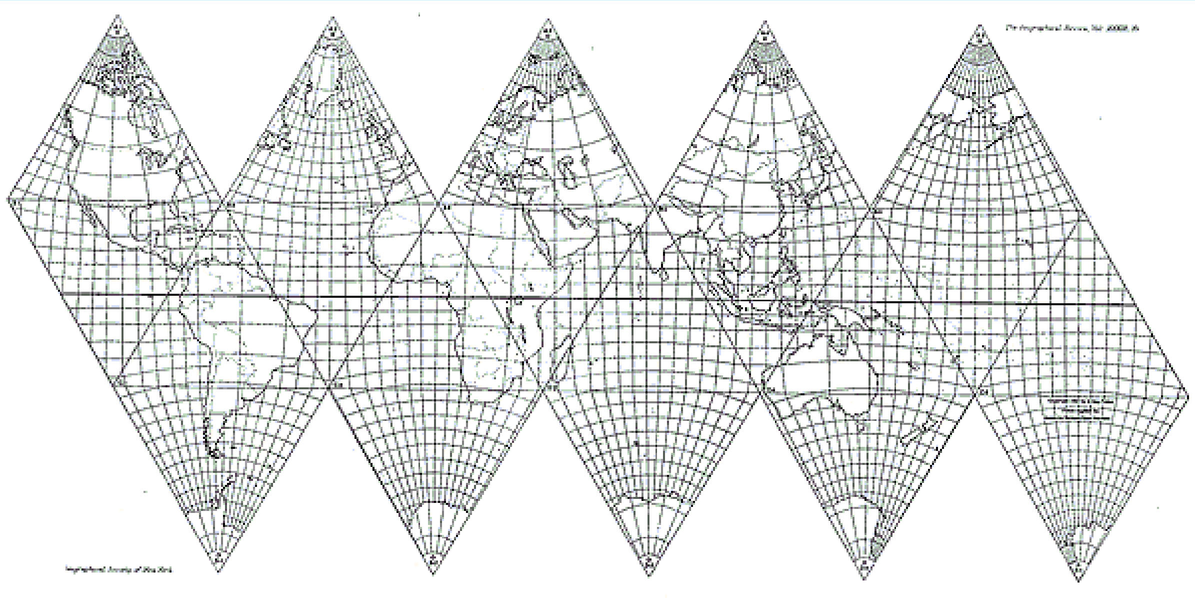

Figure 3: The world on an icosahedron by Irving Fisher, courtesy of the American Geographical Society. See (4).

In the hope that we don't continue this narrow approach to thinking about map projections, I have become interested in two projections which are available commercially in pieces (2): the Guyou by Athelstan Spilhaus made up of 32 squares (another has 72 squares); and the several variations on the icosahedron, e.g., Buckminster Fuller's Air-Ocean World, a projection made up of 20 equilateral triangles. In this paper I make use of one of the commercially available variations, a game called Flight Lines (3), and provide a reproducible variation which has been provided by the American Geographical Society (4) in Figure 3. In this form, it is possible for children to move them about and make their own versions of the world (Figure 4) - something they are asked to do in the ICA's Barbara Petchenik International Children's Map Design Competition. Also, in manipulating these maps, a number of concepts about and problems in transforming the surface of the earth to a map are made clear. This seems a good way of making ideas about projections and their use in design more accessible to young mappers.

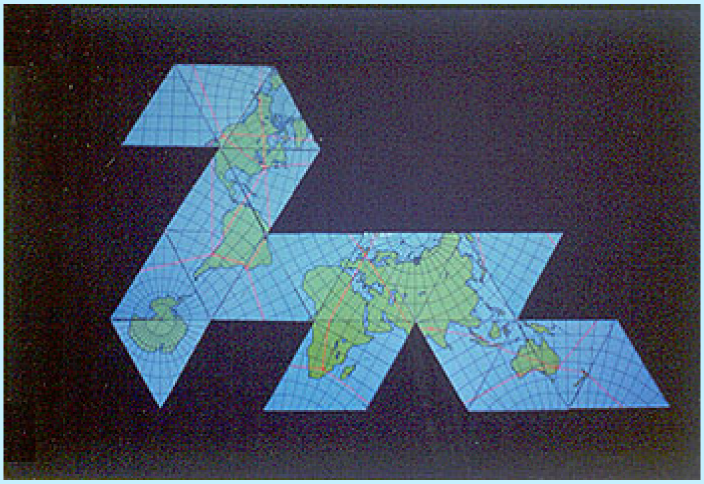

Figure 4: The "stegosaurus" projection made from two sets of Flight Lines on the icosahedron. See (3).

Getting Started

Our first question asks: "How do all these squares or triangles go together?" There are not an infinite number of combinations, only those that retain the contiguous relationships of land masses and oceans -- but even here there are quite a number.

In the simplest case, putting together such sets of figures is a test of one's knowledge of the earth's features. But once accomplished it is clear that other arrangements are possible. This comes from the realization that individual pieces can be connected in different places (three in the case of each triangle and four in the case of each square). There may also be the desire to make certain countries or continents whole instead of split or divided. The very fact that there isn't just one way to put these projections together raises questions about the very nature of map projections, i. e., the problem of trying to map the spherical surface of the earth on a flat plane. Obviously these two surfaces aren't applicable, i.e., they cannot be cut and unfolded to a flat surface like we can with solids like cones, cylinders and the Platonic solids. As a result, the earth's surface must be stretched, compressed or separated in some way. For any of these manipulative projections, these imperfections will be most obvious. Most world maps which appear to be continuous and whole hide these imperfections from the casual reader. Cartographers have mapped the pattern of distortion across most projections, so we know the pattern and that it is constant for all aspects.

While books and atlases for young students don't make much of it, any formal map projection can be produced in equatorial, polar, transverse, and oblique forms and can be centered on any point on the surface of the earth. As a result, there are an infinite number of possible forms of that projection. We can simulate this fact by constructing a projection around any one of the twenty triangles of the icosahedron or the 32 squares of the Guyou.

Many students, concerned with tests and grades, may ask another question: "Is there a correct form or, more bluntly, is there a right answer?" Of course not! The only way to determine the "right" projection is to know for what purpose it is being produced. Each may be better in solving a particular problem or illustrating a given proposition. Mappers must be tuned into this possibility and be searching for the best arrangement of a projection that will address their problem. This suggests that our teaching about map projections should always be in the context of solving problems so that manipulating the projection is seen as part of seeking a solution.

In other words, map projections are tools in geographic inquiry. Not only do they allow the display of certain information in the most useful perspective but they can also facilitate seeking a potential answer. In particular, manipulating the projection allows many general geographic propositions to be explored -- propositions such as centrality, periphery, orientation, connectivity or great circle linkages. The advantage to young students of these manipulative projections is that these explorations can be manual, improvisational, and inexpensive. Using xerox copies of a map on, for example, the icosahedron, allow many children to do these manipulations at the same time and share and compare their solutions.

Specific Activities

More specifically, there are a number of activities that children can perform with these manipulative projections that lead to other discoveries. Eight are outlined here:

1. Make a map of the world centered on one of the oceans: the Atlantic, the Pacific, the Indian, the Arctic, or the Antarctic. In comparing these five different maps, one might ask: "How many oceans are there?" Each map might support a different answer. Since there are no precise or easily defined boundaries between these water areas, it raises questions about what criteria were employed to distinguish them or whether they are just names of convenience? Many geographic generalizations are just this -- we accept their general meaning (and the errors or ambiguities that go with them) because they are useful at the level of generalization that we use them. Precise definitions may not always be necessary. But one wonders if children understand this?

2. Similarly, make a map centered on each of the continents: North America, South America, Europe, Africa, Asia, or Oceania. With these six maps in view, how many land masses are there? Is this the same as the number of continents? Again, we might try and develop some rules to support answers such as 3, 4, 5, or 6. Are these generalizations of convenience as well? Are children savvy to this?

3. Name a place at or near the center of each triangle or square and make a world map centered on one of them. For the Flight Lines icosahedron, the following are likely choices:

Yellowknife, NWT, Canada Bermuda The Galapagos Islands
Tristan de Cunha Island Mendoza, Argentina Crete
Dakar or Cape Blanc, West Africa Cape Town, South Africa Mogadiscio, Somalia
Tobolsk, Russia Beijing, China Padang, Sumatra, Indonesia
Perth, Western Australia New Caledonia Amery Ice Shelf, Antarctica
Mt. Siple, Antarctica and then four water sites in various places in the Pacific Ocean

4. From this one could construct a map illustrating some hypothetical relationship between several of these places. For example, a business might have a raw material source in one place, a manufacturing site in another (where there is an available labor supply) so that the product can be sold in a third place (a market). These locations can be determined in several graded ways: by a simple random draw from a bowl whereby names are drawn in sequence as the sites for raw materials, manufacturing center, and market area. A second way is by trying to create a logical connection between three drawn places given what knowledge students have or can find out about these places. For example, Beijing is obviously in a very large potential market. Yellowknife may have an important source of some precious metal, but we wish to provide employment for our workers in Mendoza. Your map may show the flow of materials or money within this three point system, i.e., of this international corporation.

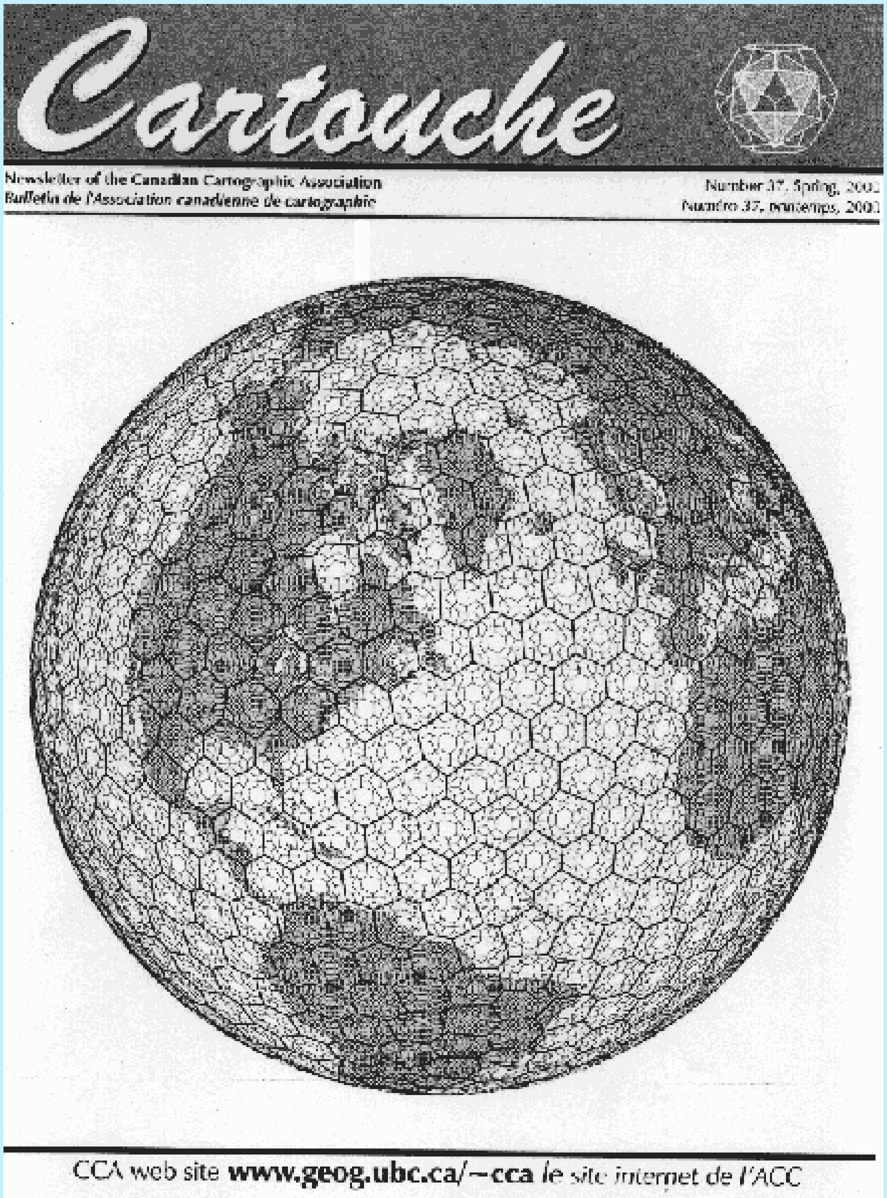

Figure 5: A "harmonic decomposition" of a dodecahedron courtesy of Hrvoje Lukatela that was reproduced on the cover of Cartouche, the newsletter of the Canadian Cartographic Association, #28, Winter 1996.

5. Mapping with these worlds divided into triangles or squares reminds one of the recent cover of Cartouche [Figure 5] and the theoretical work of Tissot who imagined the earth's surface divided into an infinite number of small circles [Robinson,1963,324f]. In projection, they are transformed in size, shape, or both. I have always felt that we should reverse the process and make a three dimensional solid out of the deformed circles. The uneven shaped and sized ellipses will produce a most dramatic deformed globe - imagine Figure 5 with highly differing hexagons. In classroom practice, this would be nearly impossible, but we might simulate this process with a simple cubic projection where the six square sides, as in Figure 6, are transformed into rectangles. Thus it seems a more accessible idea for children than the isarithmic mapping of the Tissot index values. This stretching and deforming of small areas leads nicely into an introduction to the idea of **cartograms** and the mapping of intangible dimensions of places. Obviously we could also get into the problem of scale variation by making systematic comparisons of specific distances on the globe made from these various projections.

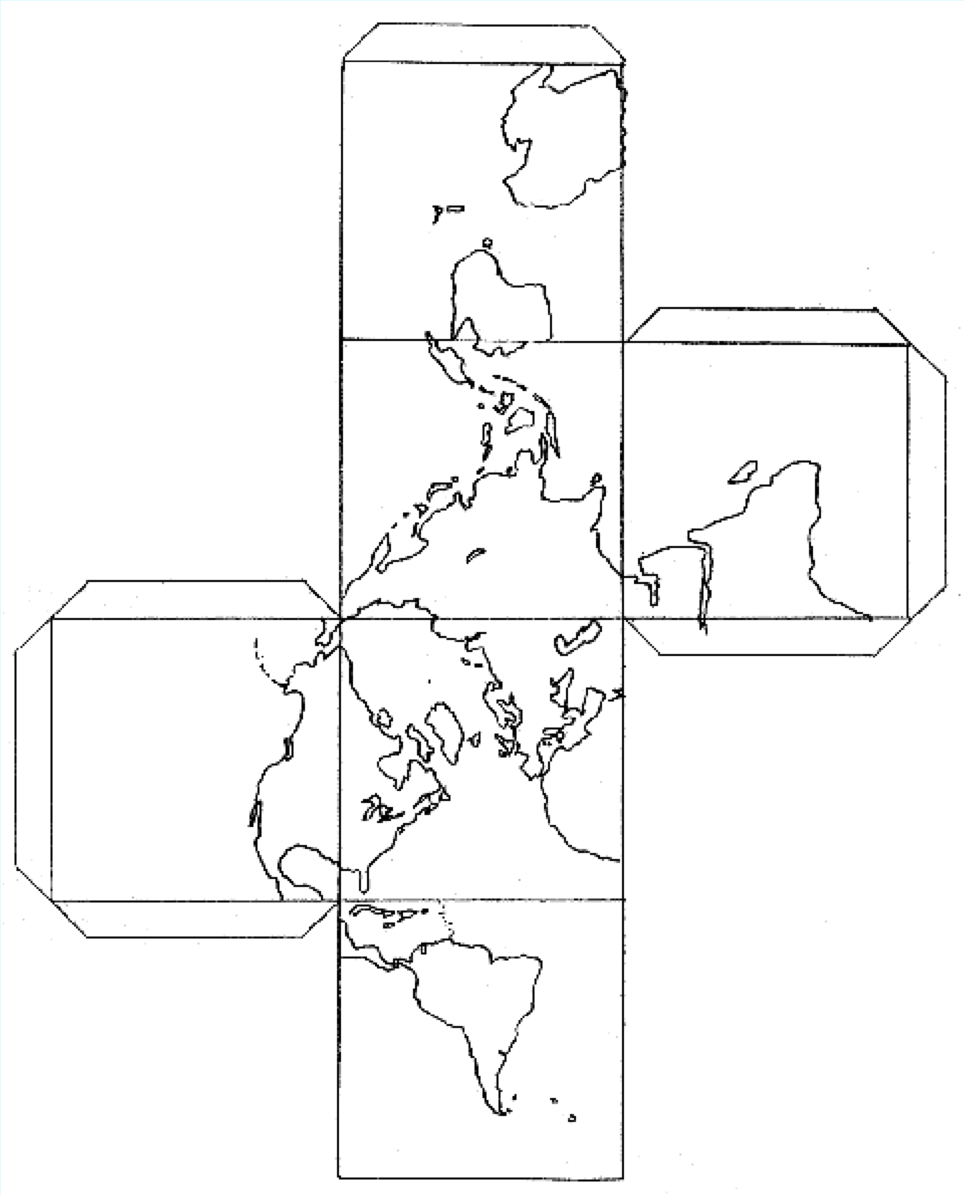

Figure 6: The projection of the earth on the sides of a cube after James R. Wray's Corners of the Earth, Association of American Geographers, Chicago, 1995.

Obviously, the icosahedron is one of the six Platonic solids -- three dimensional solids made up of equilateral plane figures, whether triangles, squares, or pentagons. All of them can be considered approximations of the globe at various levels of precision or generalization. Through central or gnomonic projection, the world can be projected onto each facet -- this is the essence of any manipulative projection. Obviously the more facets, the smaller is the coverage of each, and the better the representation. But we can never reduce the amount of distortion below some threshold, no matter what we try. The value of manipulative projections lies in making this fact apparent and to reinforce the idea that for solutions to most problems, there are always trade offs and compromises.

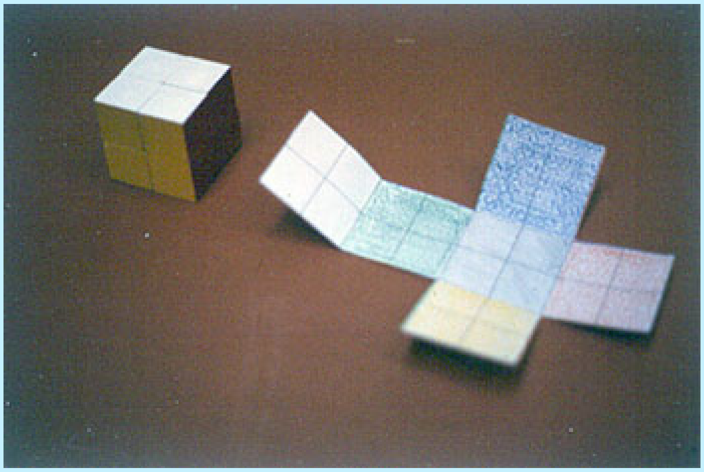

Figure 7: A colored cube and its "projection".

6. One of the simplest connections with mathematics and geometry is to cut up a piece of paper that when folded will completely enclose a pile of blocks. This enclosing represents symbolically the peeling away of the outer surface of the pile -- the same thing that map projections do with the spherical earth. We encounter this type of projection in descriptive geometry. But we don't normally think of the earth having a front, left side, top, etc. By coloring the facets of a cube we can then keep track of "sides" (Figure 7). By then adding some continents, no matter how abstract, we can give the cube pile some geographic credence. These six squares are yet another projection that can be manipulated like the 20 triangles of the icosahedron or the 32 squares of the Guyou projection.

Figure 8: A world map on the Guyot projection made up of 32 squares.

7. There is also a direct connection with algebra. Cartographers only place a few intersecting parallels and meridians on maps. In so doing, they subdivide the map into large areas which can be seen as lying in rows and columns, as in Figure 8. By labeling each column with a different consecutive letter of the alphabet and each row with a consecutive number, the map can now be searched systematically by areas defined by these two descriptors. Such alpha-numeric location systems are common to all maps but several different manipulative projections can be used to illustrate the basic principle that the smaller the alpha-numeric area (i.e., the more parallels and meridians on the map), the easier it will be to find a place although there will be more areas defined by the system. Consider, for example, the different number of places that would be found in the indexes by facet for world maps made up of 6 squares, 20 triangles, 32 squares, and 72 squares (the other Guyou projection). The logical extension of this process is to consider giving every place a unique identifier in latitude and longitude, that is by having an infinite number of parallels and meridians. In algebra, this essentially means defining locations by measuring their distance from two principal axes. Our system of parallels and meridians is one such **Cartesian coordinate system** with the **Equator** and **Prime Meridian** as the two principle axes.

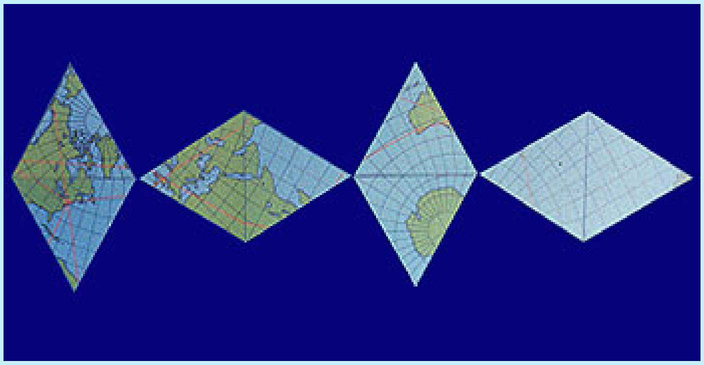

Figure 9: Eight triangles of a map on an icosahedron laid out along a great circle.

8. The shortest distance between any two places on the earth is along a **great circle**. A globe allows us to both lay out a great circle and to measure distances along it. Most world maps aren't as useful unless the two places lie along the same straight line meridian or the equator. For all others, we need at least two maps on the gnomonic projection. But we can lay out eight of the triangles of the icosahedron to form a complete great circle [Figure 9]. All of the edges of these triangles are parts of great circles. But no two consecutive edges follow the same one. On the other hand, one great circle can be laid out starting with the common edge of one pair of triangles. It continues along the perpendicular bisectors of two successive triangles laid base to base. Then it runs along another common pair of edges followed by two more perpendicular bisectors to complete the great circle. Once in place [Figure 10] we can add the other 12 triangles . I find this image, as you also might, to be a most bizarre map of the world. But after thinking about it, I realize how honest it is. We would normally encounter such a world view in an oblique aspect of a projection, as in Figure 2 where it is all neatly connected and continuous. We have "conveniently" hidden, if you will, all these interruptions by stretching the surface acro ss the open spaces.

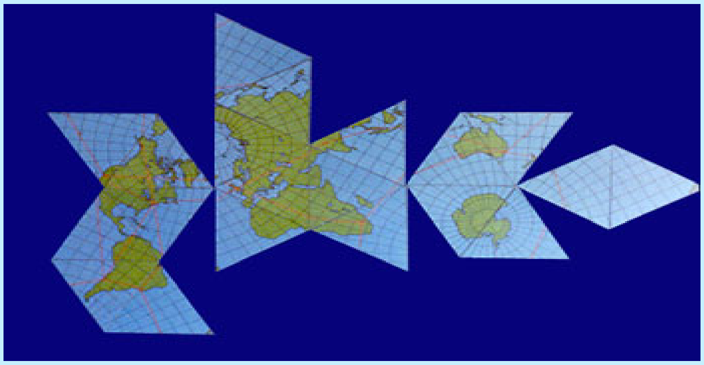

Figure 10: The remaining 12 triangles of a map on an icosahedron laid out to form a map of the world.

Obviously, these projections can be used to study the problem of measuring great distances across the earth's surface. On a world map, we can make large errors if we are unable to follow standard lines or measure places within areas of low deformation. Fortunately, the manipulative projections with 20 or more pieces have relatively low rates of deformation within each square or triangle -- the cost for this is, of course, all the interruptions. But children should be able to make reasonably accurate, and certainly consistent and thus comparable, distance measurements over long distances as long as they don't try to measure across any of the interruptions. This is an intuitive assumption but perhaps someone would like to take it on as a small research problem.

Conclusions

I have suggested how manipulative projections can provide access to at least eight different concept, skill or knowledge areas. They are:

1. Knowledge of the major features of the earth is necessary for and will come with putting the pieces together.
2. There are an infinite number of possible map projections.
3. Making a map of the world involves making concessions, i.e., accepting errors (i.e., discontinuities, interruptions, and scale variations).
4. The form, aspect, or arrangement of world map projections is under the control of the mapper and it can be modified to help address his or her communication or design problem.
5. Arranging the world map in different ways can raise questions about the nature of some common geographic terms

and generalizations.
6. Considering the earth's surface as sets of geometric figures introduces the idea of cartograms in which we change the size of each to represent less tangible aspects of those places.
7. The number of parallels and meridians on a world map can be related to ideas about Cartesian co-ordinate systems, on the one hand, and to algebra on the other.
8. The concept of great circle routes and the measurement of long distances can be demonstrated and tested with manipulative projections.

These are not the usual ideas that one sees in treatments of map projections in texts and atlases for children. I would suggest, however, that they are far more useful and accessible ideas for young mappers when asked to consider questions of world geography. Their value lies in the active, improvisational participation they require for young mappers to discover their meaning. This stands in stark contrast to the usual systematic presentation of facts about specific map projections and how they are formed mathematically. Given all this, I believe we have in manipulative projections a valuable new instrument for teaching about one of the most complex and abstract aspects of map making -- map projections -- and one that connects them to other important areas of the curricula. This is where I see making manipulative projections an attractive component of the Barbara Petchenik Map Design Competition. I say this for four reasons:

First, while some reasons are explicit, most are implicit and come with manipulating these projections. This reflects the old adage: "I hear and I forget, I see and I remember, and I do and I understand." This is, in part, how these ideas become more accessible to children.

Second, if the Petchenik Competition is increasingly placed in a context of problem solving or of examining a geographic proposition, then these projections provide built-in ways of exploring and expressing specific contest themes. In other words, map projections are tools in geographic inquiry. Not only do they allow the display of certain information in the most useful perspective but they can also in their arranging facilitate finding a potential answer. In particular, manipulating a projection allows many general geographic propositions to be explored - propositions such as centrality, periphery, specific orientation, connectivity or great circle linkages.

Third, giving young students the freedom to make their own arrangements of the world, provides them one more element of design in their problem solving or presentation.

Fourth, by demonstrating how manipulating these projections can connect with other curricula areas, the Petchenik Competition need not be a stand alone event. If teachers can connect it with other topics that they are teaching, they will be more likely to make room for it in their tight classroom schedules.

In sum, the value of manipulative projections lies in the active, improvisational participation they require for young mappers in which they can discover so many concepts and ideas. This stands in stark contrast to the usual systematic presentation of facts about specific map projections and how they are formed mathematically. Given all this, I believe manipulative projections are a valuable new instrument for teaching about one of the most complex and abstract aspects of map making - map projections - and one that connects them to other important areas of curricula.

Notes

(1) Some other unusual projections include the hyperboloid projection, essentially an interrupted orthographic produced by Erwin Raisz. It was rendered by Richard E. Harrison on the cover of Scientific American, Nov. 1975. The equal area projection with straight line parallels and meridians was described by O.S. Adams in 1945 in item #27 in Snyder and Stewart (1988). Athelstan Spilhaus' published a world ocean projection in Mapline, Sept. 1982. For variations of the equal area Sinusoidal projection see McBride & Thomas (1949), p. 45 in U.S. Dept. of Commerce Publication #245.

(2) The Guyot projection on 32 squares was available commercially from GeoLearning International Ltd., 244 North Main, P.O. Box 8711, Sheridan, WY, 82801, USA. It was sold as item #A002 under the name of GeoOdyssey game for $14.95 plus 10% shipping. The company is no longer active but stocks of this game may still be available.

(3) GeoLearning also sold a game called Flight Lines as item #006 for $5.95 plus shipping. In it the earth is projected on to ten double-sided equilateral triangles; thus to make up the icosahedron, one must purchase two sets.

(4) On January 24, 2000 the American Geographical Society granted serial rights to the reproduction of a "globe-forming map" by Irving Fisher that he first published in 1943. They request a credit line in all adaptations that might be made, See his discussions in Fisher and Miller, 1944, 92f. Afurther discussion of the use of this projection in

classrooms can be found in A Teacher's Introduction to the Barbara Petchenik International World Map Design Competition at .

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